Refutation of Comments by Bruhn et al.

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The following is a refutation of the paper: Gerhard W. Bruhn, Friedrich W. Hehl, and Arkadiusz Jadczyk, *Comments on "Spin Connection Resonance in Gravitational General Relativity"* [arxiv.org/pdf/0707.4433v1].

Detailed points of refutation

The comments assert that Cartan geometry is "undefined", which alone should be enough to arouse suspicion. Cartan geometry is standard mathematics, and is described in numerous textbooks (for example, [2]).

The problem with their use of Eq. ((6)) is that this is the Einsteinian form without torsion, and not the second Bianchi identity of Cartan geometry (the correct second Bianchi identity is Eq. (2), below).

It has been shown in UFT Paper 88 (aias.us) that the first and second Bianchi identities are, in indexless notation:

$$D \wedge T := R \wedge q \tag{1}$$

and

$$D \wedge (D \wedge T) := D \wedge (R \wedge q) \ . \tag{2}$$

The traditional second Bianchi identity [2]:

$$D \wedge R = 0 \tag{3}$$

is a special case of Eq. (2).

The authors of that comment then seem to accept the fact that

$$d \wedge R^a_b = R^a_c \wedge \omega^c_b - \omega^a_c \wedge R^c_b \tag{4}$$

is a rewriting of Eq. (3) in the form:

$$d \wedge R^a_b = j^a_b \ . \tag{5}$$

The second basic error made by the authors is to then assert that Eq. (4) does not imply

$$d \wedge \tilde{R}^a_b = \tilde{R}^a_c \wedge \omega^c_b - \omega^a_c \wedge \tilde{R}^c_b \quad . \tag{6}$$

(This error has also been refuted in detail in UFT Paper 89).

To see that this is an error, first write out Eq. (4) in full:

$$(d \wedge R^a_b)_{\mu\nu\rho} \;=\; R^a_{c\mu\nu} \wedge \omega^c_{\rho b} - \omega^a_{\rho c} \wedge R^c_{b\mu\nu} \quad . \tag{7}$$

The Hodge dual of R^a_b is defined as [2]:

$$\tilde{R}^a_{b\mu\nu} \;=\; \frac{1}{2}|g|^{\frac{1}{2}}\,\bar{\epsilon}^{\,\mu\nu\rho\sigma}\,R^a_{b\rho\sigma} \quad , \tag{8}$$

where

$$\epsilon_{\mu_1\mu_2\ldots\mu_n} = |g|^{\frac{1}{2}}\,\bar{\epsilon}_{\mu_1\mu_2\ldots\mu_n} \tag{9}$$

is defined by [2]:

$$|g| = \|g_{\mu\nu}\| \quad . \tag{10}$$

Apply the Hodge dual (8) to both sides of Eq. (4):

$$d \wedge (\epsilon R) = (\epsilon R) \wedge \omega - \omega \wedge (\epsilon R) \; . \tag{11}$$

Then use the metric compatibility condition [2]:

$$D_\mu g_{\nu\rho} = 0 \; , \tag{12}$$

to find that

$$d \wedge (\epsilon R) = \epsilon d \wedge R \; . \tag{13}$$

Therefore,

$$d \wedge \tilde{R} = \tilde{R} \wedge \omega - \omega \wedge \tilde{R} \; . \tag{14}$$

Q.E.D.

All of the comments concerning the index a have already been refuted. The meaning of the a index was first made clear as far back as 1992 [3], and published material about that index is available in approximately 25 properly refereed journals (Omnia Opera section of aias.us). A detailed discussion is also available in UFT Paper 89.

We are apparently told next that the traditional Bianchi identity [2]:

$$D \wedge R = 0 \tag{15}$$

is not the same as its own tensor formulation:

$$D_\sigma R^\kappa_{\mu\nu\rho} + D_\sigma R^\kappa_{\rho\mu\nu} + D_\sigma R^\kappa_{\nu\mathrm{p}\mu} = 0 \ \ . \tag{16}$$

If there are any readers left who continue to take these individuals seriously, the present author points out the textbooks again, which show that the form equations of Cartan geometry all have their tensor equivalents.

The tensor formulation (16) can be rewritten as [2]:

$$D^\mu G_{\rho\mu} = 0 \ , \tag{17}$$

where $G_{\rho\mu}$ is the Einstein tensor.

The Einstein field equation is then

$$D^\mu G_{\rho\mu} = k D^\mu T_{\rho\mu} \ \ , \tag{18}$$

where

$$T_{\rho\mu} = T_{\mu\rho} \tag{19}$$

is the symmetric canonical energy-momentum tensor of Noether, and where k is the Einstein constant.

Therefore, Eq. (18) can equally well be written as

$$\left(D_\sigma R^\kappa_{\mu\nu\rho} + D_\sigma R^\kappa_{\rho\mu\nu} + D_\sigma R^\kappa_{\nu\rho\mu}\right) = k\left(D_\sigma N^{\mathrm{k}}_{\mu\nu\rho} + D_\sigma N^{\mathrm{k}}_{\rho\mu\nu} + D_\sigma N^{\mathrm{k}}_{\nu\rho\mu}\right) \ \ , \tag{20}$$

which is

$$D \wedge R^a_b = k D \wedge N^a_b \ \ . \tag{21}$$

Q.E.D.

We are next told that "... the metric component g^{00} of the Minkowski metric is not a constant function (sic) of x^i (sic)." On the contrary, the Minkowski metric is

$$g^{\mu\nu} = g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{22}$$

and, therefore, $g^{00} = 1$. (23)

This is a number, (i.e., 1), and as such is independent of x^i, a component of a complete vector field. The critics have again contrived an "error" where none exists.

Finally, we are told that there exist no resonance solutions to the equation:

$$\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} - \frac{1}{r}\phi = -\frac{\rho}{\epsilon_0} \quad , \tag{24}$$

with $\rho = \rho(0)\cos(\kappa_r r)$. (25)

On the contrary, if we make the change of variable [1]:

$$\kappa_r r = \exp(i\kappa_r R) \, , \tag{26}$$

then Eq. (24) becomes

$$\frac{d^2\phi}{dR^2} + \kappa_r^2\phi = \frac{\rho(0)}{\epsilon_0}\,\mathrm{Re}\left(e^{2i\kappa_r R}\cos\left(e^{i\kappa_r R}\right)\right), \tag{27}$$

which has resonance solutions. Q.E.D. (Please note that the equation of ECE theory that leads to Eq. (24) is Eq. (1).)

Also, please note that Eq. (28), below, is Eq. (9.32) of Paper 63, which is the resonance equation for the variable omega, in spherical coordinates:

$$\frac{\partial^2\phi}{\partial r^2} + \left(\frac{2}{r} + \omega_r\right)\frac{\partial\phi}{\partial r} + \frac{\phi}{r^2}\left(2r\omega_r + r^2\,\partial\frac{\omega_r}{\partial r}\right) = -\frac{\rho}{\epsilon_0} \quad , \tag{28}$$

and when the spin connection is defined as

$$\omega_r = \omega_{0r}^2 - 4\beta\log_e r - \frac{4}{r} \quad , \tag{29}$$

Eq. (28) takes the form:

$$\frac{\partial^2\phi}{\partial r^2} + 2\beta\frac{\partial\phi}{\partial r} + \omega_0^2\phi = -\frac{\rho}{\epsilon_0} \quad , \tag{30}$$

which is a resonance equation. Q.E.D.

References

[1] M. W. Evans, *Generally Covariant Unified Field Theory* (Abramis Academic, 2005 to 2009), Volumes 1 to 5 (consisting of UFT Papers 1-89 on www.aias.us).

[2] S. M. Carroll, *Space-time and Geometry, an Introduction to General Relativity* (Addison Wesley, New York, 2004), Chapter 3. (In addition, S. M. Carroll, *Lecture Notes on General Relativity*, are available at [arxiv.org/abs/gr-qc/9712019].)

[3] M. W. Evans, Physica B, 182, 227, 237 (1992). □